Instituto de Física
Universidade Federal Fluminense

Eletromagnetismo

Newton Mansur

Equações de Maxwell

Lei de Gauss

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

Lei de Ampère

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{d\vec{E}}{dt}$$

Lei de Faraday

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$

Equações de Maxwell

Lei de Faraday $$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \vec{\nabla} \times -\frac{d\vec{B}}{dt} = -\frac{d}{dt}\vec{\nabla} \times \vec{B}$$

$$\nabla^2\vec{E} - \vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{E}) = -\frac{d}{dt}\vec{\nabla} \times \vec{B}$$

Lei de Gauss $$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$ *para* $\rho = 0$ $$\vec{\nabla} \cdot \vec{E} = 0$$

Lei de Ampère $$\vec{\nabla} \times \vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{d\vec{E}}{dt}$$

para $\vec{J} = 0$ $$\vec{\nabla} \times \vec{B} = \mu_0\epsilon_0\frac{d\vec{E}}{dt}$$

$Equações\ de\ Maxwell$

$$\nabla^2\vec{E} = -\mu_0\epsilon_0\frac{d^2\vec{E}}{dt^2}$$

$Em\ 1\ dimensão$

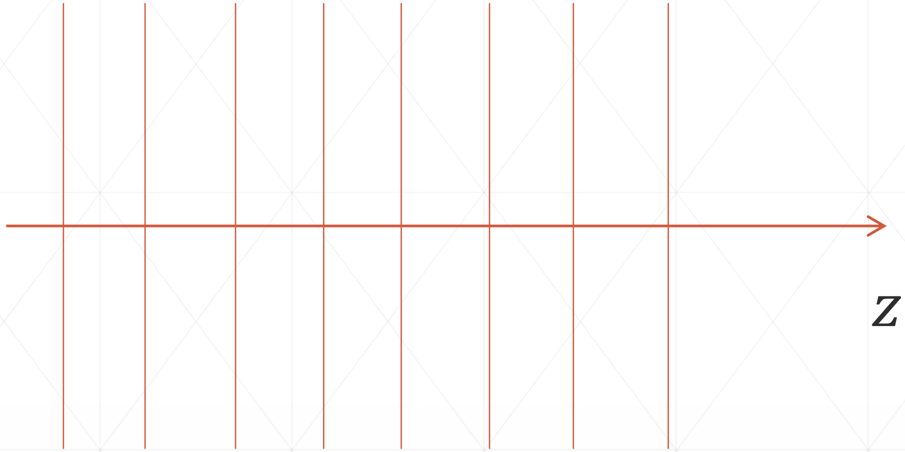
$$\frac{d^2E_x(z,t)}{dz^2} = -\mu_0\epsilon_0\frac{d^2E_x(x,t)}{dt^2} = -\frac{1}{v^2}\frac{d^2E_x(x,t)}{dt^2}$$

$$v = \sqrt{\frac{1}{\mu_0\epsilon_0}} = 2{,}998x10^8\frac{m}{s} = c$$

$$\nabla^2 \vec{E} = \mu\epsilon \frac{d^2\vec{E}}{dt^2}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\left(E_x\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z\right) = \mu\epsilon \frac{d^2}{dt^2}\left(E_x\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z\right)$$

Em 1 dimensão



$$\left(\frac{\partial^2}{\partial z^2}\right)\left(E_x\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z\right) = -\mu\epsilon \frac{d^2}{dt^2}\left(E_x\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z\right)$$

$$v = \sqrt{\frac{1}{\mu\epsilon}}$$

$$\vec{E} = \vec{E}_0 e^{j(\omega t - \beta z)}$$

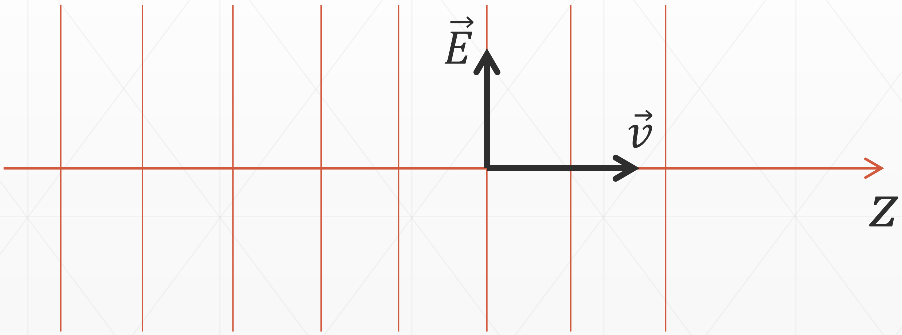
$$\left(\frac{\partial^2}{\partial z^2}\right)\left(E_x\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z\right) = -\mu\epsilon\frac{d^2}{dt^2}\left(E_x\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z\right)$$

$$\vec{E} = \vec{E}_0 e^{j(\omega t - \beta z)} + \vec{E}_1 e^{j(\omega t + \beta z)}$$

$$-\beta^2\left(E_x\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z\right) = -\mu\epsilon\omega^2\left(E_x\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z\right)$$

$$\frac{\omega^2}{\beta^2} = \frac{1}{\mu\epsilon} \qquad \frac{\omega}{\beta} = \sqrt{\frac{1}{\mu\epsilon}} = v \qquad \vec{\nabla}\cdot\vec{E} = 0 \qquad \frac{\partial E_z}{\partial z} = 0 \qquad \beta E_z = 0 \qquad E_z = 0 \qquad \vec{v} \perp \vec{E}$$

$$\left(\frac{\partial^2}{\partial z^2}\right)\left(E_x\hat{a}_x + E_y\hat{a}_y\right) = -\mu\epsilon\frac{d^2}{dt^2}\left(E_x\hat{a}_x + E_y\hat{a}_y\right)$$



$Em\ 3\ dimensões$ $\vec{E} = \vec{E}_0 e^{j(\omega t - \vec{k}.\vec{r})}$ $\vec{\nabla} \cdot \vec{E} = 0$ $\vec{k} \cdot \vec{E} = 0$

$Onda\ transversal\ onde$ $\vec{k} \perp \vec{E}$ $k = \beta$

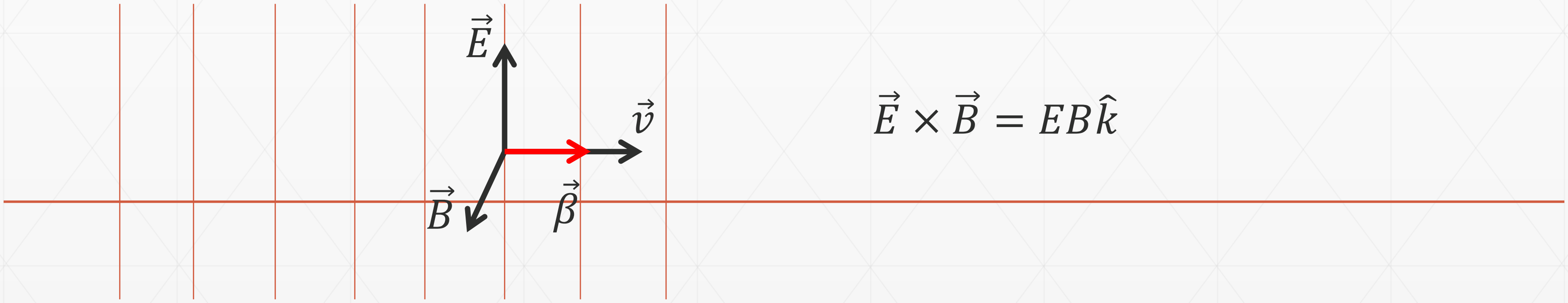
$\vec{E}$ $é\ a\ polarisação\ da\ onda$

$\hat{k}$ $é\ a\ direção\ de\ propagação\ da\ onda$

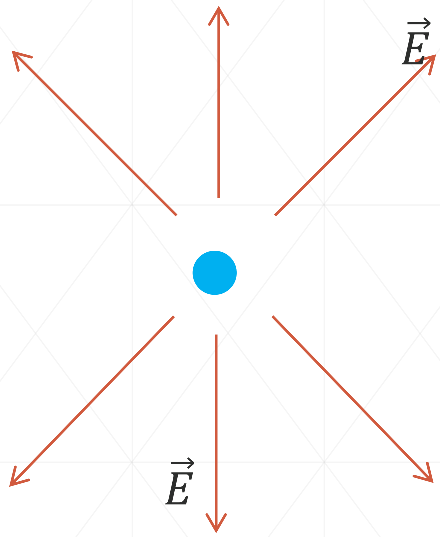
$$\nabla^2 \vec{B} = \mu\epsilon \frac{d^2\vec{B}}{dt^2} \qquad \vec{B} = \vec{B}_0 e^{j(\omega t - \vec{k}.\vec{r})}$$

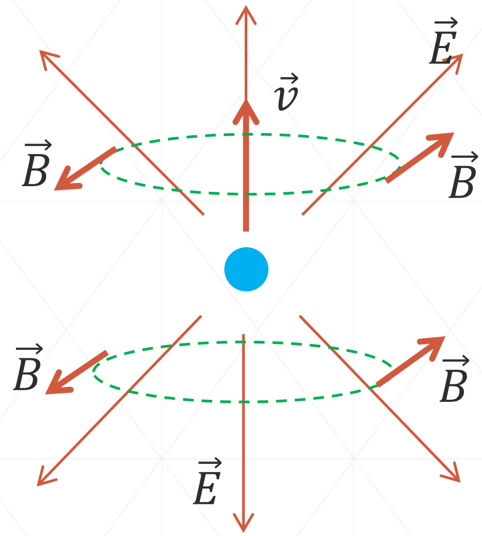
$$\vec{\nabla} \times \vec{B} = \mu\epsilon \frac{d\vec{E}}{dt} \qquad \vec{k} \times \vec{B} = -\mu\epsilon\omega\vec{E} \qquad \vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt} \qquad \vec{k} \times \vec{E} = -\omega\vec{B}$$

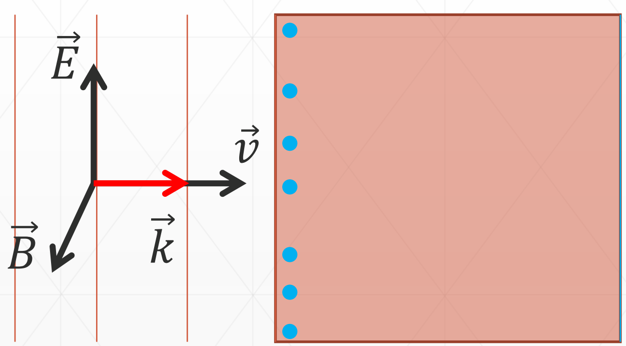
$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{k} \cdot \vec{B} = 0 \qquad \vec{k} \perp \vec{B} \qquad \vec{E} \perp \vec{B}$$

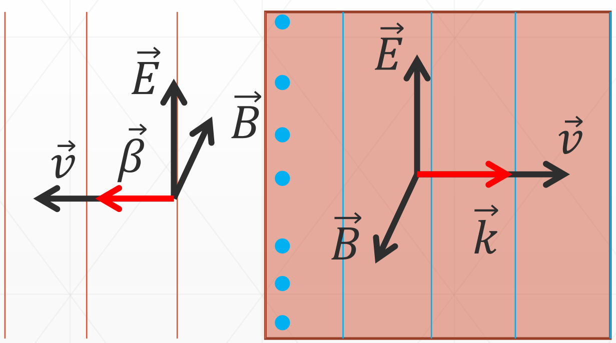


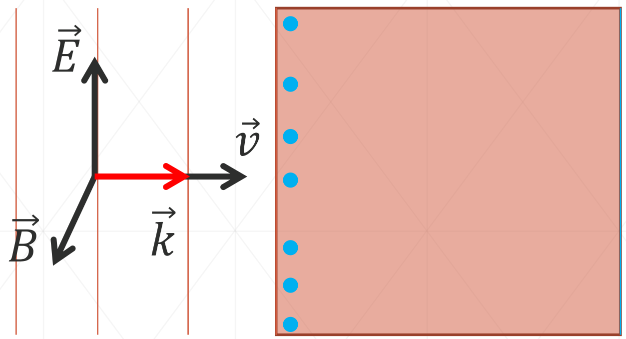
$$\vec{E} \times \vec{B} = EB\hat{k}$$

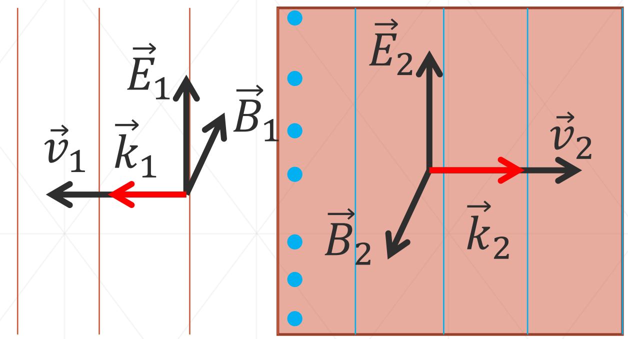

$\vec{E}$
$\vec{E}$


$\vec{E}$
$\vec{v}$
$\vec{B}$
$\vec{B}$
$\vec{B}$
$\vec{B}$
$\vec{E}$


$\vec{E}$
$\vec{v}$
$\vec{B}$
$\vec{k}$


$\vec{E}$
$\vec{B}$
$\vec{v}$
$\vec{\beta}$
$\vec{E}$
$\vec{v}$
$\vec{B}$
$\vec{k}$

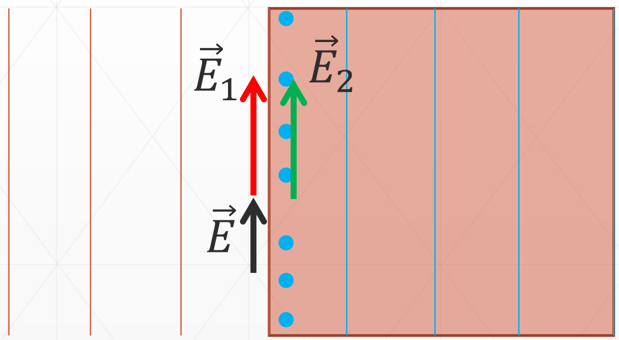


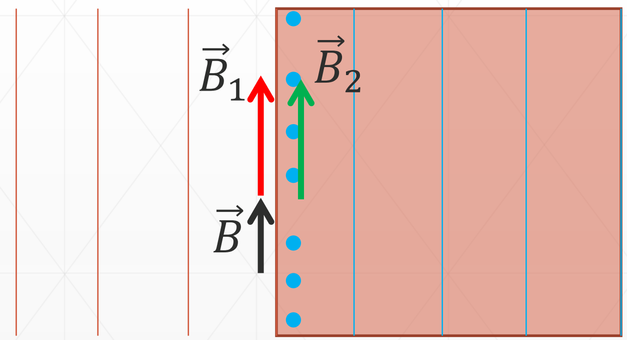


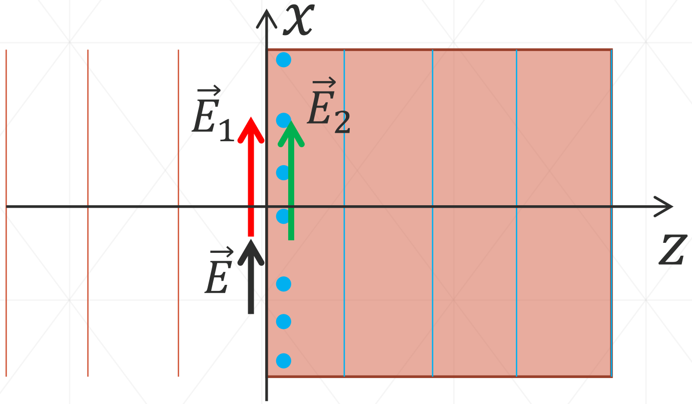
$$\vec{E} = \vec{E}_0 e^{j(\omega t - \vec{k}.\vec{r})}$$

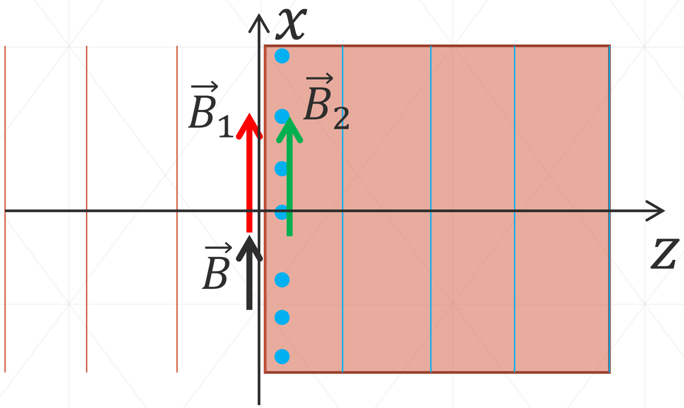
$$\vec{E}_1 = \vec{E}_{01} e^{j(\omega t + \vec{k}_1.\vec{r})}$$

$$\vec{E}_2 = \vec{E}_{02} e^{j(\omega t + \vec{k}_2.\vec{r})}$$









$$\vec{E} = \vec{E}_0 e^{j(\omega t - \beta z)}$$

$$\vec{E}_1 = \vec{E}_{01} e^{j(\omega t + \beta_1 z)}$$

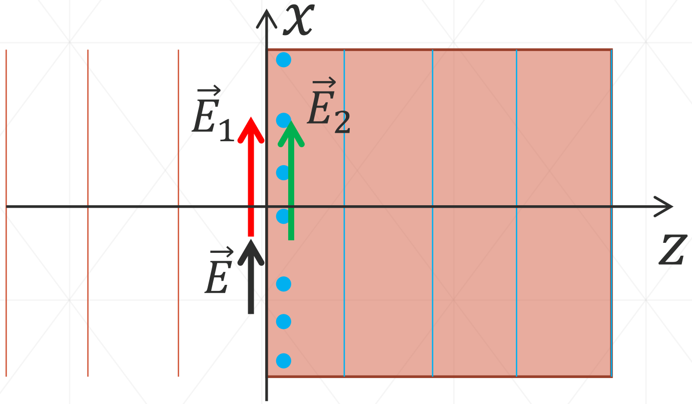
$$\vec{E}_2 = \vec{E}_{02} e^{j(\omega t - \beta_2 z)}$$

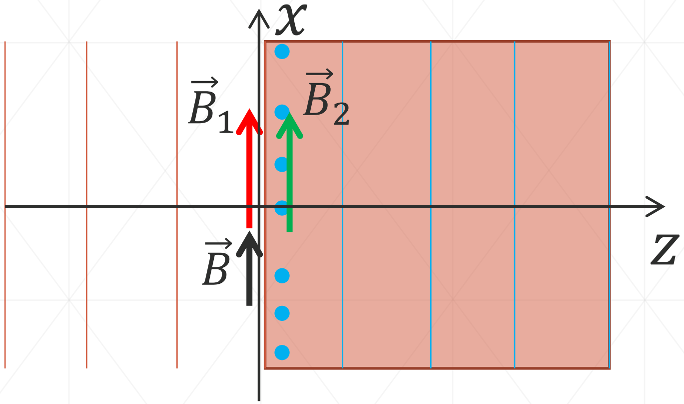
$$\vec{k} \times \vec{E} = -\omega \vec{B} \qquad kE = \omega B \qquad B = \frac{k}{\omega} E = \frac{1}{v} E$$

$$\vec{B} = \frac{1}{v_1} \vec{E}_0 e^{j(\omega t - \beta z)}$$

$$\vec{B}_1 = -\frac{1}{v_1} \vec{E}_{01} e^{j(\omega t + \beta_1 z)}$$

$$\vec{B}_2 = \frac{1}{v_2} \vec{E}_{02} e^{j(\omega t - \beta_2 z)}$$





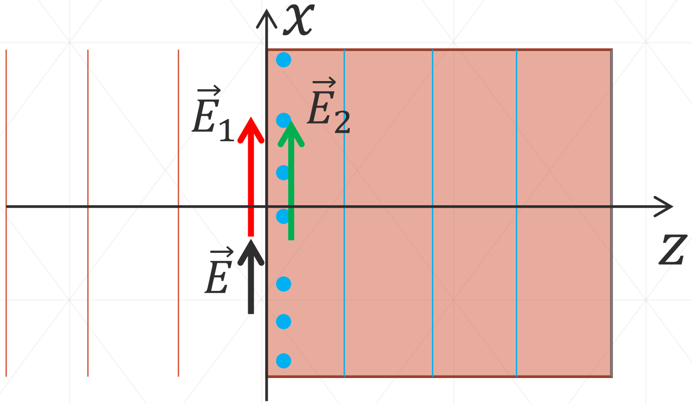
$$\epsilon_1 E_{1n} = \epsilon_2 E_{2n} \qquad B_{1n} = B_{2n}$$

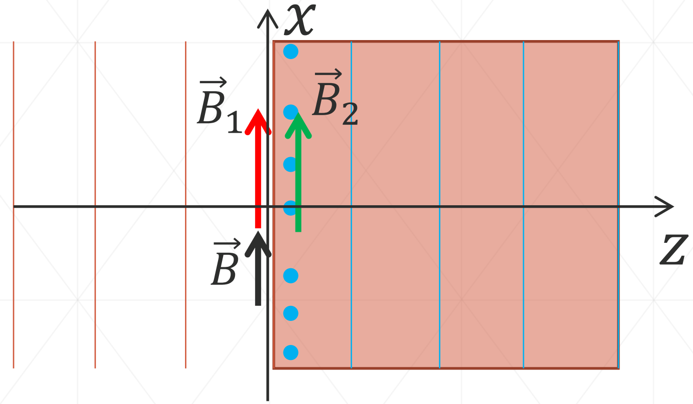
$$E_{1t} = E_{2t} \qquad E_t + E_{1t} = E_{2t}$$

$$\frac{1}{\mu_1} B_{1t} = \frac{1}{\mu_2} B_{2t} \qquad \frac{1}{\mu_1}(B_t + B_{1t}) = \frac{1}{\mu_2} B_{2t} \qquad \frac{1}{\mu_1}\left(\frac{1}{v_1} E_t - \frac{1}{v_1} E_{1t}\right) = \frac{1}{\mu_2}\frac{1}{v_2} E_{2t}$$

$$H = \frac{1}{\mu} B \qquad H = \frac{1}{\mu} B = \frac{1}{\mu}\frac{1}{v} E \qquad Hl \to I \qquad El \to V \qquad \frac{El}{Hl} = \frac{V}{I} \qquad \frac{E}{H} = \mu v \to R$$

$$\frac{E}{H} = \mu v = \eta \quad \to \quad Impedância\ intrinseca\ do\ meio$$





$$\frac{1}{\mu_1}\left(\frac{1}{v_1}E_t - \frac{1}{v_1}E_{1t}\right) = \frac{1}{\mu_2}\frac{1}{v_2}E_{2t} \qquad \frac{1}{\eta_1}E_t - \frac{1}{\eta_1}E_{1t} = \frac{1}{\eta_2}E_{2t} \qquad E_t + E_{1t} = E_{2t}$$

$$E_{01} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}E_0 \qquad E_{02} = \frac{2\eta_2}{\eta_2 + \eta_1}E_0$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1}$$

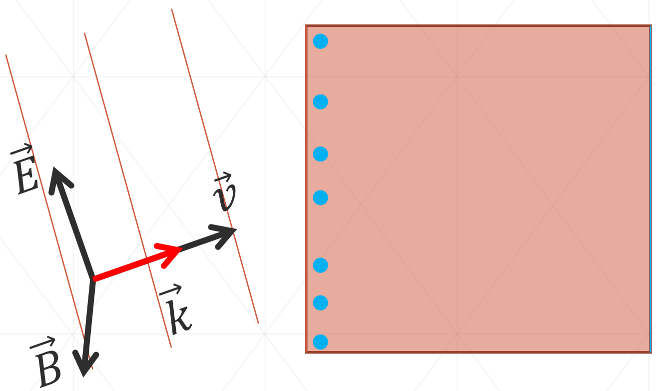
$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \rightarrow \textit{Coeficiente de reflexão}$$

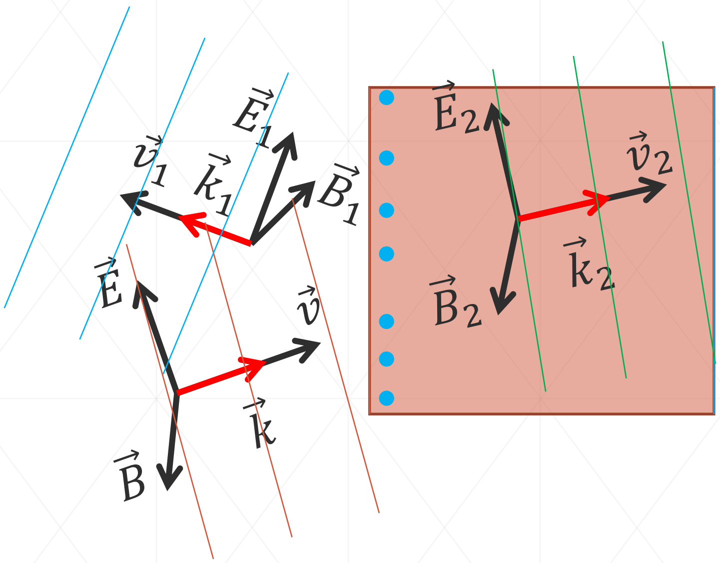
$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} \rightarrow \textit{Coeficiente de transmissão}$$

$$\Gamma + 1 = \tau$$

$$-1 \leq \Gamma \leq 1$$

$$\vec{E} = \vec{E}_0 e^{j(\omega t - \vec{k}.\vec{r})} \qquad \vec{B} = \vec{B}_0 e^{j(\omega t - \vec{k}.\vec{r})}$$



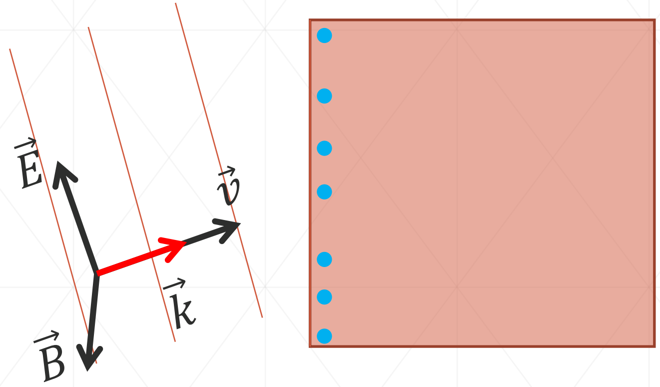


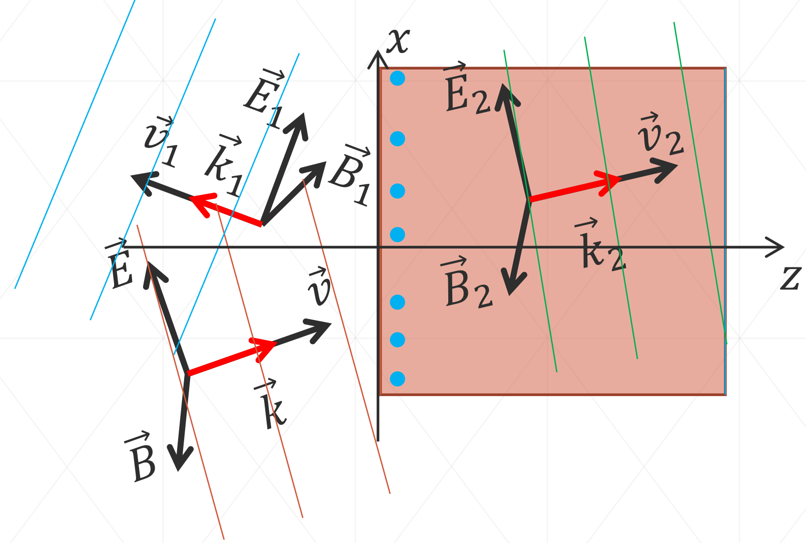
$$k^2 = k_x{}^2 + k_y{}^2 + k_z{}^2 = \omega^2 \mu \epsilon \qquad \vec{k}.\vec{r} = k_x x + k_y y + k_z z = cte$$

$$\vec{k} \times \vec{E} = \omega \mu \vec{H} \qquad \eta = \mu v \qquad \hat{k} \times \vec{E} = \eta \vec{H}$$

$$\vec{k} \times \vec{H} = \omega \epsilon \vec{E}$$

$$\vec{k} \cdot \vec{H} = 0 \qquad \vec{k} \cdot \vec{E} = 0$$





$$\vec{E} = \vec{E}_0 e^{j(\vec{k}.\vec{r} - \omega t)} = \vec{E}_0 e^{j(k_x x + k_y y + k_z z - \omega t)}$$

$$\vec{E}_1 = \vec{E}_{01} e^{j(\omega t + \vec{k}_1.\vec{r})} = \vec{E}_{01} e^{j(k_{1x} x + k_{1y} y + k_{1z} z - \omega t)}$$

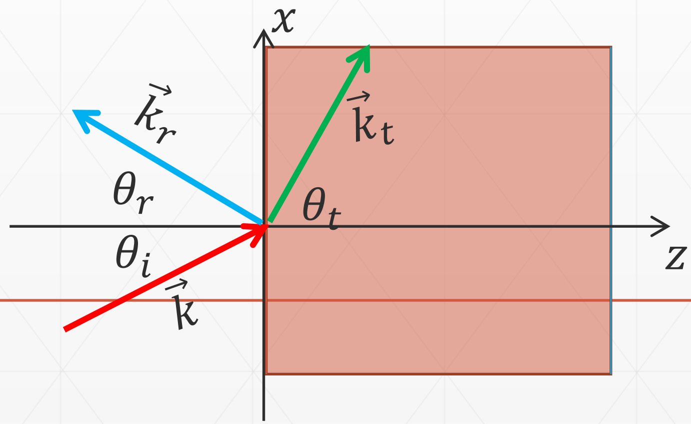
$$\vec{E}_2 = \vec{E}_{02} e^{j(\omega t - \vec{k}_2.\vec{r})} = \vec{E}_{02} e^{j(k_{2x} x + k_{2y} y + k_{2z} z - \omega t)}$$

Para $z = 0$ *e para qualquer* x, y

$$\vec{E}_t + \vec{E}_{1t} = \vec{E}_{2t}$$

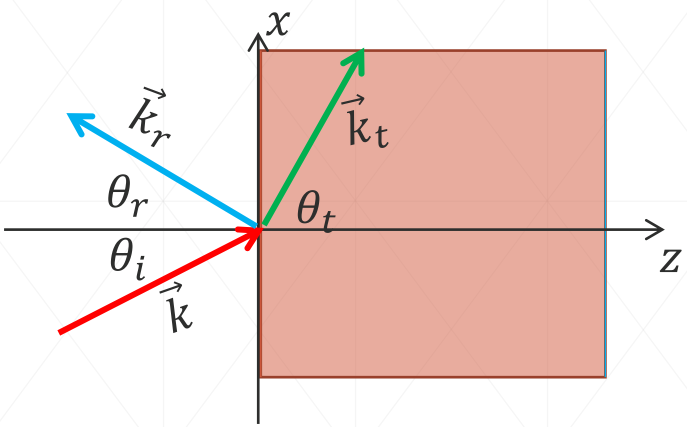
$$k_x = k_{1x} = k_{2x}$$

$$k_y = k_{1y} = k_{2y}$$



$$k\ sen\theta_i = k_r sen\theta_r = k_t sen\theta_t$$

$$k = k_r = \beta = \frac{\omega}{v} \qquad \theta_i = \theta_r$$



$$k\ sen\theta_i = k_r sen\theta_r = k_t sen\theta_t$$

$$k = k_r = \beta = \frac{\omega}{v} \qquad \theta_i = \theta_r$$

$$\frac{sen\theta_t}{sen\theta_i} = \frac{k_t}{k} = \frac{\omega/k}{\omega/k_r} = \frac{v_1}{v_2} = \frac{c/v_2}{c/v_1} = \frac{n_2}{n_1}$$

Lei de Snell

Linhas de transmissão

$$V(z,t) = {V_0}^+ e^{-\alpha z}\cos(\omega t - \beta z) + {V_0}^- e^{\alpha z}\cos(\omega t + \beta z)$$

$$I(z,t) = {I_0}^+ e^{-\alpha z}\cos(\omega t - \beta z) + {I_0}^- e^{\alpha z}\cos(\omega t + \beta z)$$

$$Z_0 = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} \qquad \gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\lambda = \frac{2\pi}{\beta} \qquad v(\omega) = \frac{\omega}{\beta(\omega)} \qquad z(\omega) = \frac{1}{\alpha(\omega)} \rightarrow \textit{Decaimento em } \frac{1}{e}$$

$$I(z,t) = \frac{{V_0}^+}{Z_0} e^{-\alpha z}\cos(\omega t - \beta z) - \frac{{V_0}^-}{Z_0} e^{\alpha z}\cos(\omega t + \beta z)$$

Linhas de transmissão

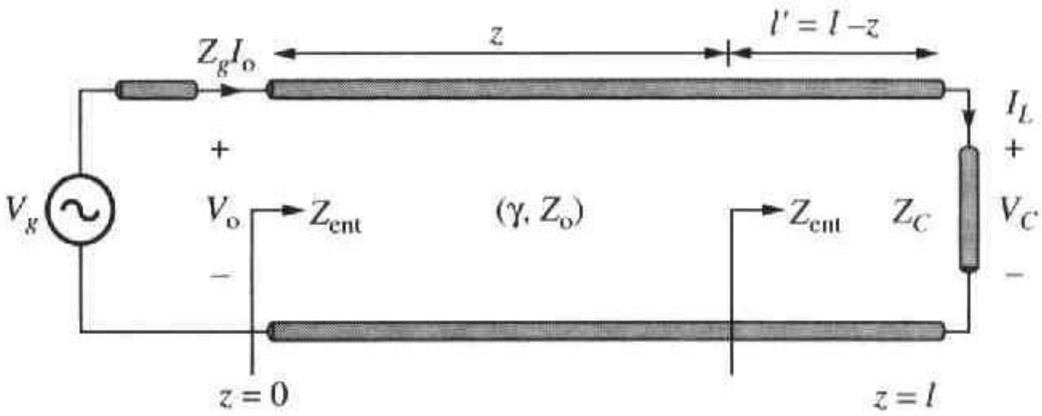
$$V(z,t) = {V_0}^+ e^{-\alpha z} \cos(\omega t - \beta z) + {V_0}^- e^{\alpha z} \cos(\omega t + \beta z)$$

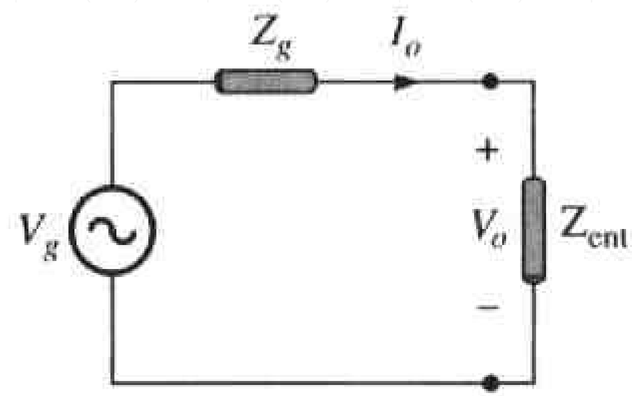
$$I(z,t) = \frac{{V_0}^+}{Z_0} e^{-\alpha z} \cos(\omega t - \beta z) - \frac{{V_0}^-}{Z_0} e^{\alpha z} \cos(\omega t + \beta z)$$

$z = 0$ $\quad V_0 = V(z = 0)$ $\quad I_0 = I(z = 0)$

$${V_0}^+ = \frac{1}{2}(V_0 + Z_0 I_0) \qquad {V_0}^- = \frac{1}{2}(V_0 - Z_0 I_0)$$

Linhas de transmissão





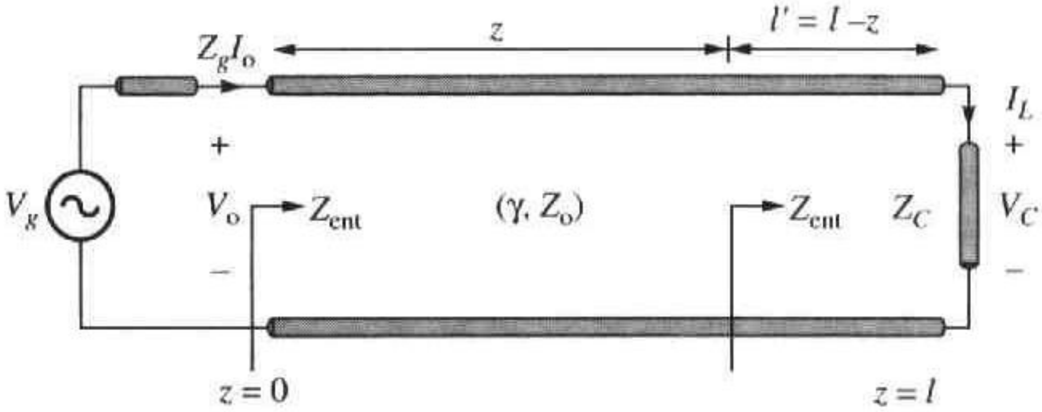
$$V_0^{+} = \frac{1}{2}(V_0 + Z_0 I_0) \qquad V_0^{-} = \frac{1}{2}(V_0 - Z_0 I_0)$$

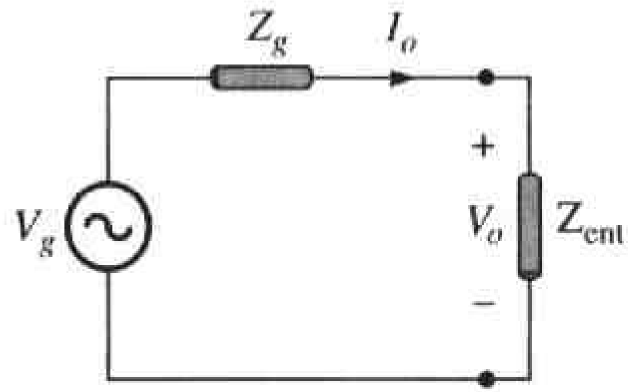
$$V_0 = \frac{Z_{ent}}{Z_{ent} + Z_g} V_g \qquad I_0 = \frac{V_g}{Z_{ent} + Z_g}$$

$$z = l \qquad V_C = V(z = l) \qquad I_C = I(z = l)$$

$$V_0^{+} = \frac{1}{2}(V_C + Z_0 I_C) e^{\gamma l} \qquad V_0^{-} = \frac{1}{2}(V_C - Z_0 I_C) e^{-\gamma l}$$

Linhas de transmissão





$$V_0^{\ +} = \frac{1}{2}(V_C + Z_0 I_C)e^{\gamma l} \qquad V_0^{\ -} = \frac{1}{2}(V_C - Z_0 I_C)e^{-\gamma l}$$

$$Z_{ent} = \frac{V_S(z)}{I_S(z)} = \frac{Z_0\left(V_0^{\ +} + V_0^{\ -}\right)}{V_0^{\ +} - V_0^{\ -}} = Z_0 \frac{(Z_C + Z_0 tanh\gamma l)}{(Z_o + Z_C tanh\gamma l)}$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \frac{V_0^{\ -} e^{\gamma l}}{V_0^{\ +} e^{-\gamma l}} = \frac{Z_C - Z_0}{Z_C + Z_0} = \Gamma_C$$